School of Computing, Engineering, and the Built Environment
Edinburgh Napier University

PHD STUDENT PROJECT

Funding and application details

Funding status: Self funded students only

Application instructions:
Detailed instructions are available at https://blogs.napier.ac.uk/scebe-research/available-phd-student-projects/

Prospective candidates are encouraged to contact the Director of Studies (see details below) to discuss the project and their suitability for it.

Project details

Supervisory Team:

- Director of Study: Chris Guiver (Email: C.Guiver@napier.ac.uk)
- 2nd Supervisor: Yuyang Zhou

Subject Group: Engineering & mathematics

Research Areas: Control engineering, Applied Mathematics, Engineering Mathematics, Mathematical Modelling

Project Title: Projects in mathematical systems and control theory

Project description:

Mathematical systems and control theory “represents an attempt to codify, in mathematical terms, the principles and techniques used in the analysis and design of control systems” [1]. It concerns itself with ``the basic theoretical principles underlying the analysis of feedback and the design of control systems [and] differs from the more classical study of dynamical systems in its emphasis on inputs (or controls) and outputs (or measurements)” [2].

Control systems and feedback loops are ubiquitous in science and engineering, and arise in traditional areas from aerospace control, manufacturing and process control, through to robotics, electrical power systems, systems biology, and therapeutics. The modern study of control systems traces its roots back to the industrial revolution and the principles of feedback control are at the heart of the flyball governor which was used regulate pressure in steam engines and facilitate their safe operation. Nowadays, the design and deployment of control systems has been fantastically successful, with 31 real-world success stories described in [3]. Novel and emerging applications where control systems play an essential role range from the control of autonomous vehicles, smart grids and devices, through to epidemiology [4].

Mathematical control theory is broad and makes connections to numerous other mathematical areas, including:

- analysis and operator theory via the control of infinite-dimensional systems — such as delay- or partial differential- equations from either a PDE or abstract functional analytic perspective [5];
- calculus of variations and optimisation via optimal control theory and, increasingly prevalently, stochastic optimal control theory [6];
- numerical analysis and numerical linear algebra via singular value- or Krylov subspace- methods for model order reduction [7].
- The present project welcomes applications for doctoral research in a wide range of control theoretic settings. Potential PhD projects are offered in themes of expertise of the supervisor, including:
- Topics in nonlinear stability theory --- analysing and predicting the long-term behaviour of various classes of control systems specified by nonlinear differential or difference equations. This theme focusses primarily on (controlled) systems of differential equations.
- Regularity properties of classes of linear operators arising commonly in control theory (namely, Hankel and Toeplitz operators). It is well-known that these operators admit representations in terms of the Fourier transform and multiplication by certain classes of complex-valued analytic functions (often called symbols), and this relationship shall be further explored. This theme focusses primarily on functional analysis and operator theory.
- Topics in so-called positive and monotone control systems [8], roughly dynamical systems which evolve on a positive cone, reflecting the typical requirement that state (and possibly other) variables are naturally nonnegative in some sense. This theme is broad but combines elements from linear algebra and dynamical systems theory. There is scope within this theme for applications of control theory to theoretical ecology and mathematical biology; see, for example [9].
- Applications of control theory to energy storage and renewable energy generation. This theme focusses primarily on optimisation and optimal control applications.

The projects offered are mathematical, although there is ample scope for more theoretical or more applications-focussed projects, which may contain a large numerical/computational component, depending on applicant and the overall direction of the research. Further, the second supervisor may change depending on the theme chosen. Please also note that prior experience of mathematical control theory is not a requirement --- strong mathematics knowledge and experience is essential, however. Perspective applicants are encouraged to contact the Supervisor before submitting their applications, highlighting which of the above areas are of most interest. Applications should make it clear the project you are applying for and the name of the supervisors.

References:

[1] P. Falb. Methods of Algebraic Geometry in Control Theory: Part I, Birkhäuser, 1990.

[2] I. Karafyllis & Z.-P. Jiang. Stability and Stabilization of Nonlinear Systems, Springer-Verlag, 2011

[3] T. Samad & A. E. Annaswamy (eds.). The Impact of Control Technology, 2nd edition, IEEE Control Systems Society, 2014.

[4] C. Beck et al. Special Section on Mathematical Modeling, Analysis, and Control of Epidemics, SIAM J. Control Optim., 2022, 60(2): Si-Sii

[5] M. Tucsnak & G. Weiss. Observation and control for operator semigroups, Birkhäuser, 2009.

[6] D. Liberzon. Calculus of variations and optimal control theory: a concise introduction, Princeton University Press, 2012.

[7] A. C. Antoulas. Approximation of large-scale dynamical systems, SIAM, 2005.

[8] D. Angeli & E.D. Sontag. Monotone control systems, IEEE Trans. Automat. Control, 2023, 48(10), 1684-1698.

[9] C. J. Edholm, C. Guiver, R. Rebarber, B. Tenhumberg & S. Townley. Stabilization by Adaptive Feedback Control for Positive Difference Equations with Applications in Pest Management, SIAM J. Control Optim,. 2022, 60(4)

Candidate characteristics

Education:

A first-class honours degree, or a distinction at master level, or equivalent achievements in mathematics or another cognate discipline

Subject knowledge:

- With a fundamental knowledge of at least one of the project themes mentioned above

Essential attributes:

- Concepts and techniques in mathematics for theme of interest (analysis, differential equations, dynamical systems, linear algebra, optimisation)
- Competent in learning new mathematics from a range mathematical areas
- Systems of differential equations

Desirable attributes:

- Programming (Matlab/Python/C/C++/Fortran);
- Mathematical systems and control theory.