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| **Department** | School of Engineering and the Built Environment |
| **Supervisors** | Chris Guiver and Stathis Tingas |
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| **Project Title** | Model order reduction of (bio-)chemical kinetics and epidemiological systems |

## PROJECT DESCRIPTION

Mathematical modelling is an essential ingredient for fundamental research of large and complex multi-scale systems in a diverse range of fields from astrophysics and chemical kinetics, to economics, electronics and logistics. Such systems are inherently highly complex, typically described by hundreds, if not thousands, of variables and processes. The sheer size of these systems often renders obtaining meaningful physical insight intractable, or at the least highly tedious. Moreover, in control theory, designing controllers based on these large models is also challenging, since the controller may have a comparable order (“model size”) to that of the to-be-controlled system.

To tackle the large size and high complexity of such systems, model order reduction is usually pursued. Model order reduction essentially seeks to approximate a given model with one of lower complexity, which is ideally qualitatively and quantitatively close to the original. Model order reduction provides both computational cost reduction and insight into key facets or aspects of the original model, as well as a simpler framework for controller design.

There are numerous model order reduction methods, including those based on singular values (principal component analysis), Proper Orthogonal Decomposition, Dynamic Mode Decomposition, Petrov-Galerkin methods, and timescale exploitation methods, such as singular perturbation methods. Each method has its theoretical and computational advantages and disadvantages.

For example, on the one hand, in mathematical control theory, where dynamical systems usually have notions of input variables and output variables, so-called balanced truncation is a popular model order reduction method. It is based on truncating the state-space in accordance with the relative size of singular values of an operator associated with the original control system, and produces a reduced-order model which approximates the original input-output relationship. Balanced truncation methods have many appealing properties, such as known a priori error bounds, and its variants are known to preserve structural (qualitative) properties of the original model --- a desirable feature.

On the other hand, geometric Singular Perturbation (GSP) theory was developed in the 1970s and enabled the identification of dynamical structures in the phase space and the exploitation of their properties. Techniques like the Intrinsic Low Dimensional Manifold (ILDM), the Zero Derivative Principle (ZDP), the Computational Singular Perturbation (CSP) theory and others have been used in a wide range of different fields, such as chemical kinetics, infectious disease transmission, systems biology, atmospheric science and many others.

Mathematically, model order reduction methods combine tools from numerical linear algebra, numerical analysis and analysis. Their study is inherently both analytical and also numerical, as developing algorithms which effectively compute reduced-order models is essential.

In the current project, a range of model order reduction methods, and their combinations, will be studied and developed in order to develop an applicable and advanced, yet well-founded, mathematical framework for model order reduction particularly of complex nonlinear systems appearing in models for combustion, (bio)-chemical kinetics and disease transmission. The supervisors have broad expertise in these areas. Tools from perturbation theory and asymptotic analysis shall also be employed. The project is theoretical but contains a large numerical component since the developed framework will be tested against actual modern detailed models. There is scope for more theoretical or more computational-based projects.

Perspective applicants are encouraged to contact the Supervisor before submitting their applications. Applications should make it clear the project you are applying for and the name of the supervisors.

**Academic qualifications**
A first degree (at least a 2.1) ideally in Applied Mathematics/Computational Physics/Mathematical Biology with a good fundamental knowledge of mathematical modelling and analysis of dynamical systems.

**English language requirement**
IELTS score must be at least 6.5 (with not less than 6.0 in each of the four components). Other, equivalent qualifications will be accepted. Full details of the University's policy are available online.

**Essential attributes:**

- Experience of fundamental mathematical modelling of linear or non-linear systems.
- Competent in programming (Matlab/Python/C/C++/Fortran)
- Knowledge of linear algebra, systems of ODEs.
- Good written and oral communication skills
- Strong motivation, with evidence of independent research skills relevant to the project
- Good time management

**Desirable attributes:**
Knowledge of mathematical modelling of systems of multiple timescales, control theory, model reduction, dynamic mode decomposition, singular perturbation theory.

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| **Indicative Bibliography** | 1. L. Fortuna, G. Nunnari and A. Gallo. (1992). Model order reduction techniques with applications in electrical engineering, Springer-Verlag, London.<br>2. P.J. Schmid. (2010). Dynamic mode decomposition of numerical and experimental data. Journal of Fluid Mechanics 656.1 (2010): 5–28.<br>3. Banasiak, J., & Lachowicz, M. (2014). Methods of small parameter in mathematical biology. Springer International Publishing<br>4. Vynnycky, E., & White, R. (2010). An introduction to infectious disease modelling. OUP Oxford.<br>5. Verhulst, F. (2005). Methods and applications of singular perturbations. Springer.<br>6. Obinata, G., & Anderson, B. D. (2001). Model reduction for control system design. Springer-Verlag, London. |
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| **Enquiries** | For informal enquiries about this PhD project, please contact Dr Chris Guiver via c.guiver@napier.ac.uk |
| **Web page** | https://www.napier.ac.uk/research-and-innovation/research-degrees/application-process |